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Security Brokerage Markets under Price Uncertainty*

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This paper develops a model of security broker behavior under price uncertainty. The model examines the process of matching orders and the determinants of equilibrium brokerage commission rates. Institutional arrangements, search efficiency, execution costs, volume, risk, and the unit price of the security are shown to affect equilibrium brokerage commission rates. Some stylized facts of security brokerage are explained. *Journal of Economic Literature* Classification Numbers: D83, G0. © 1992 Academic Press, Inc.

I. INTRODUCTION

This paper develops a model of security broker behavior under price uncertainty. It examines the process of matching security orders and the determinants of brokerage commission rates. The paper contributes to the security brokerage literature in the following ways: (1) It provides a formal analysis of the search and match behavior of brokers in security trading. (2) It provides a theoretical framework for determining the effects of important financial and economic factors on brokerage commission

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rates and for evaluating the efficiency of existing brokerage systems. (3) It spells out the interactions between brokerage and dealer service markets and explores the relationship between brokerage commissions and bid–ask spreads. (4) It compares alternative brokerage structures and analyzes their impacts on trading volume and transaction costs.

The main findings are: (1) Brokerage commission rates are inversely related to brokers' search efficiency, the order size, the unit price of the security, and the number of competing market makers and are positively related to order execution costs and security risk. (2) Brokerage commission rates are lower when bid–ask spreads are smaller. (3) In-house execution services increase both brokers' profits and search costs. Interdealer brokerage reduces the dispersion of dealers' price quotations and transaction costs while increasing trading volume.

Previous studies of this subject have focused on the pricing of brokerage services (West and Tinic, 1971; Friend and Blume, 1973; Epps, 1976; Hamilton, 1976; Edmister, 1978; Tinic and West, 1980; Blum and Lewellen, 1983), but detailed analyses are few. In this paper, we provide a detailed analysis of the determination of equilibrium brokerage commission rates by examining the search behavior of security brokers.

There are a number of studies of the roles of brokers and dealers in search economies, but the focus of this paper is quite different. Whereas Garbade and Silber (1976) and Garbade (1978) examine the effect of search on price dispersion in government securities markets, this paper focuses on order matching and the determination of equilibrium brokerage commissions. Our model considers the uncertainties faced by each broker regarding the number of customers as well as order flows. Kane and Marks (1990) investigate newsletter and fund delivery systems and conclude that the newsletter is a superior market-timing information delivery system. We emphasize the role of brokers in bringing together buyers and sellers of securities. Yavas (1992) examines why dealers emerge in some markets and brokers in others. He finds that a brokerage system will emerge in a market where search is relatively inefficient and costly. Unlike Yavas (1992), we model security trading in a market system with both dealers and brokers and explore the relationship between brokerage and dealer markets. In a general equilibrium setting focusing on the welfare impact, Peck (1990) shows that a market-maker system may have advantages over an auction market with round-to-round market clearing. As does Peck (1990), we characterize uncertainty about demanders and suppliers of securities in a stochastic framework, but whereas customers in the Peck model trade directly with a single market maker, we incorporate brokers in the transaction and allow brokers and customers to trade with more than one dealer. Another distinction between Peck's paper and ours is that we explicitly consider brokers' reactions to the bid–ask spread choices of dealers.

Since brokerage services are related to dealer services, our discussions of security brokerage naturally draw on the market microstructure literature. The models of dealer behavior developed by Stoll (1978), Ho and Stoll (1980, 1981, 1983), and Yavas (1991) illuminate the linkage between brokers and dealers. This paper extends their analyses to consider the responses of dealers to changes in the brokerage market conditions.

The remainder of this paper is organized as follows. Section II develops a search model for a security broker. Section III derives the equilibrium conditions for brokerage services. Section IV examines the relationship between brokerage and dealer service markets. Section V applies the analysis to some stylized facts of brokerage services. Section VI considers the effects of transaction and security characteristics on the brokerage commission rate. Finally, Section VII presents a conclusion.

II. A MODEL OF SECURITY BROKERAGE

In this section, we first analyze the process of order matching and then determine brokers' optimal level of search. There are three types of market participants: investors (buyers or sellers), brokers, and dealers (market makers). Brokers are agents of investors, who process the public's orders to buy and sell securities. For this service a commission is charged. Dealers are individuals or firms in the securities business acting as principals. They are traders who make markets for certain securities.

Buyers and sellers of securities communicate their orders either to brokers or to dealers. They can submit market orders, limit orders, or other types of orders (e.g., stop-loss orders). In the process of buying or selling securities, they incur trading costs, which include commissions, bid–ask spreads, and other transaction costs (e.g., cost of waiting). Dealers provide immediacy, and they cover the cost of immediacy by purchasing securities at (lower) bid prices and selling at (higher) ask prices.

Brokers supply investors with a variety of services, the most basic of which is linking investors to the securities markets.[1] Brokers keep track of unfilled bids and offers and, at the appropriate time, complete transactions (Tinic and West, 1979). Acting as the focal point for buy and sell orders, brokers reduce the cost of information and waiting time. In addition, brokers facilitate the execution of orders that require secret negotiations and extensive search, and they solicit orders to fulfill outstanding tenders.

Like dealers, brokers reduce the transaction cost of securities trading, and they smooth prices. However, brokers act strictly as agents for customers whereas dealers trade as principals for their own accounts and assume the attendant price risk. Nevertheless, by handling limit orders,

[1] In addition to facilitating trades, brokers often offer investment advice, periodic statements, custodial service, loans, and investment banking or other financial services.

brokers submit quotes that constitute part of the market bid and ask prices.

In developing the model, we assume the following:

(A1) Individual investors may trade securities with brokers' assistance or directly with dealers.

(A2) Dealers' bid and ask prices follow a probability distribution that is common knowledge to all market participants, but is beyond brokers' control.

(A3) The time horizon is one period, during which fundamental economic conditions do not change. Current prices and other public information are accessible to all market participants but only at a cost.

To avoid complications, we do not provide an explicit treatment of private information. It is possible that security price and volume may reflect this type of information or that brokers may obtain information through their search. Although we do not specifically address the issue of private firm information, introducing this complication will not change our qualitative results.

(A4) There is more than one dealer in a security.

(A5) Brokers are expected profit maximizers (risk neutral) but dealers and investors are risk averse.

Since the broker serves as an agent of investors, it is reasonable to assume that he/she is an expected profit maximizer. In contrast, dealers trade for their accounts and assume the risk of security holding. Furthermore, investors' portfolio decisions are exogenously determined in our model, and we simply follow the convention that investors are risk averse.

(A6) The return from a broker's search efforts increases at a decreasing rate.

(A7) The probability of attracting orders is positively related to a broker's search efforts and negatively related to the commission rate. Also, the probability of attracting buy orders decreases with the expected ask price while the probability of attracting sell orders increases with the expected bid price.

(A8) A broker's revenue from trading a security is independent of the revenue from trading another security. For simplicity, each customer's order is assumed to be for one share.

(A9) Each broker begins the period with a certain number of unexecuted orders.

Most of the above assumptions are imposed for tractability.

1. *The Process of Order Match*

Before modeling a broker's search behavior, we begin with a formal analysis of the order-matching process. The main issues are how brokers match buy and sell orders and the factors that affect brokers' ability to perform this function.

Consider a potential buyer waiting to invest. This particular buyer may submit a limit order that specifies the maximum price at which the order may be executed. Alternatively, the buyer may simply wait until he/she is informed by a broker of an acceptable (reservation) price and then place a market order. In either case, the probability of consummating the transaction depends on the probability distribution of the ask prices, which include dealers' quotes and the prices of the limit sell orders maintained by brokers in a given period. Denote $F(V_a)$ as the cumulative frequency distribution function of ask prices V_a and δ_p as the probability that a particular V_a is the minimum ask price. The probability that a purchase order is executed at the lowest possible price V_a is

$$\delta_p = 1 - [1 - F(V_a)]^{n_a} \tag{1}$$

where n_a is the number of ask prices (for a given security) occurring in a given period. For limit orders, this particular V_a has to be less than or equal to the specified limit order price. For market orders, V_a has to be less than or equal to the investor's reservation price. The term $[1 - F(V_a)]^{n_a}$ is the probability that the minimum of the n_a ask prices searched is greater than V_a. Thus, $1 - [1 - F(V_a)]^{n_a}$ is the probability that, given the n_a ask prices searched, there is at least one price that will be less than or equal to the limit order or reservation price.[2]

In priniciple, a broker can monitor price movements for a sufficiently long time to find a reasonably low price for the buyer. However, a broker may not wish to do so, because the monitoring cost can be prohibitive. A broker's optimal level of service will be determined by a trade-off between the cost and the benefit of monitoring and waiting.

The total expected number of orders matched for a given broker depends also on the probability that the broker will attract purchase orders. Denote P_p as the probability that a broker attracts a purchase order. This probability can be affected by the broker's search for buyers, S_p, and by the commission rate. The broker's search S_p consists mainly of costs of advertisement and time spent in contacting customers and providing them with market information. Let c_a be the commission rate expressed as a fraction of ask price paid by buyers. From (A7) the effect of search on P_p is positive and that of c_a is negative; that is, $\partial P_p/\partial S_p > 0$ and $\partial P_p/\partial c_a < 0$.

The expected number of matches can be expressed as

$$M_a = \delta_p(N_p P_p + \bar{z}_p), \tag{2}$$

[2] Assume that ask prices are uniformly distributed between zero and one with a unit of scale, say, 100, such that 0.5 is equal to 50 in actual value. Then, $F(V_a) = \int_0^{V_a} dx = V_a$ and $[1 - F(V_a)]^{n_a} = [1 - V_a]^{n_a}$. If the buyer specifies a limit order price equal to 50, then the probability of match, δ_p, is equal to 0.9375 if the limit (or reservation) price is obtained after four searchers (observations).

where N_p is the total number of purchase orders introduced into the markets during a time period and $\bar{z}_p$ is the number of purchase orders held by a broker at the beginning of the period. Given (A8), M_a represents the supply of brokerage service to securities buyers in terms of the number of units, e.g., shares. Both the probability of match and the expected number of purchase orders consummated depend on the extent of search, commission rates, available ask quotes, and investors' reservation prices. Intuitively, one would expect that a larger number (and greater dispersion) of available ask quotes will allow a broker to execute purchase orders at more favorable prices and, therefore, to increase the probability of order match. Similarly, the higher a buyer's reservation price, the higher the probability for a broker to match the order. Furthermore, we expect from (2) that the number of order executions is positively related to the probability of match, δ_p. Also, given (A7), the number of order executions should be affected by a broker's search effort and commission rate. Lemma 1 summarizes these ideas.

LEMMA 1. *The probability of matching purchase orders is positively related to available ask quotes in a given period and to buyers' reservation prices. The expected number of purchase orders consummated is positively related to the probability of order match and to the level of broker search and negatively related to commission rates.*

Proof. It is easy to show that $\partial\delta_p/\partial n_a > 0$, $\partial\delta_p/\partial V_a > 0$, $\partial M_a/\partial\delta_p > 0$, $\partial M_a/\partial S_p = \partial M_a/\partial P_p \cdot \partial P_p/\partial S_p > 0$, and $\partial M_a/\partial c_a = \partial M_a/\partial P_p \cdot \partial P_p/\partial c_a < 0$. ■

Following the same line of reasoning, we can specify the supply of brokerage service to securities sellers as

$$M_b = \delta_s(N_s P_s + \bar{z}_s), \tag{3}$$

where

$$\delta_s = 1 - G(V_b)^{n_b} \tag{4}$$

and

δ_s = the probability of finding at least a bid price that is greater than or equal to the seller's specified limit order or reservation price,
G = the cumulative frequency distribution function of bid prices, and
V_b = the maximum bid price.

The variables P_s, N_s, $\bar{z}_s$, and n_b, where the subscripts s and b refer to sell orders and bid prices, respectively, are analogous to the variables defined for purchase orders. $G(V_b)^{n_b}$ indicates the probability that each of the bid prices searched is lower than a particular bid price V_b, and $1 - G(V_b)^{n_b}$ is the probability that at least one bid price is higher than or equal to a particular V_b.[3] The value of V_b must be greater than or equal to the limit sell order price or the seller's reservation price.

A larger number (greater dispersion) of bid quotes tends to increase the chance of executing sell orders at more favorable prices. Also, the lower the seller's reservation price, the higher the likelihood of executing the seller's order. A higher probability of order match naturally increases the number of order executions. Furthermore, given (A7), the number of sell order executions will be affected by a broker's search for sellers, S_s, and by the commission rate, c_b. Lemma 2 summarizes these ideas.

LEMMA 2. *The probability of matching sell orders is positively related to available bid quotes in a given period and negatively related to sellers' reservation prices. The expected number of sell orders consummated is positively related to the probability of order match and to the level of broker search and negatively related to commission rates.*

Proof. It can be easily shown that $\partial\delta_s/\partial n_b > 0$, $\partial\delta_s/\partial V_b < 0$, $\partial M_b/\partial\delta_s > 0$, $\partial M_b/\partial S_s = \partial M_b/\partial P_s \cdot \partial P_s/\partial S_s > 0$, and $\partial M_b/\partial c_b = \partial M_b/\partial P_s \cdot \partial P_s/\partial c_b < 0$. ■

2. *The Broker's Optimization Problem*

A broker pays the direct costs of processing orders and receives a commission after an order is executed. A broker also incurs search costs that are not directly related to order processing. A risk-neutral broker maximizes his/her expected profit by choosing an appropriate search level and a commission rate. The expected profit from the transactions involving one particular security can be written as[4]

$$\pi_j = (c_a\underline{V}_a - \sigma_a)M_a + (c_b\underline{\underline{V}}_b - \sigma_b)M_b - \sum_k \gamma_k S_k, \qquad k = \text{p,s,b,a}, \quad (5)$$

where

π_j = the expected profit from transactions in security j,
c_a = the commission rate charged on the purchase order,

[3] The distribution function G need not be the same as F. The shape of the distribution functions for bids and asks may depend on dealers' inventory positions. Also, n_b is not necessarily equal to n_a since the number of limit sell order prices is not necessarily equal to the number of limit purchase order prices.

[4] For simplicity, the fixed cost term has been dropped.

σ_a = the purchase order costs associated with arranging trades and with recording and clearing a transaction,

c_b = the commission rate charged on the sell order,

σ_b = the sell order costs associated with trading, recording, and clearing,

$\underline{V}_a$ = the expected minimum ask price,

$\underline{V}_b$ = the expected maximum bid price,

γ_k = the unit cost of search for purchasers (k = p), sellers (k = s), bids (k = b), or asks (k = a), and

S_k = the extent of search for purchasers, sellers, bids, or asks.

Before deriving the broker's optimality conditions, it is necessary that we examine the effect of search on the expected minimum ask and maximum bid. We show below that brokerage search results in more favorable prices for buyers and sellers.

PROPOSITION 1. *The broker search for bid and ask quotes affects expected transaction prices. A greater search effort results in a lower expected minimum ask price $\underline{V}_a$ and a higher expected maximum bid price $\underline{V}_b$.*

The proof of Proposition 1 is given in the Appendix. ■

The effect of search on $\underline{V}_a$ and $\underline{V}_b$ is intuitively clear. In any given period, there is a distribution of V_a or V_b. The greater the search effort, the lower the expected minimum ask price and the higher the expected maximum bid price.

The broker's problem is to determine the search effort and a commission rate to maximum his/her expected profit. The optimality conditions are

$$(c_a\underline{V}_a - \sigma_a)N_p\delta_p \frac{\partial P_p}{\partial S_p} = \gamma_p \tag{5a}$$

$$\begin{aligned}(c_a\underline{V}_a - \sigma_a)N_p\delta_p \frac{\partial P_p}{\partial \underline{V}_a}\frac{\partial \underline{V}_a}{\partial S_a} \\ - (c_a\underline{V}_a - \sigma_a)(N_pP_p + \bar{z}_p)[1 - F(V_a)]^{n_s}\ln(1 - F(V_a))\frac{\partial n_a}{\partial S_a} \\ = \gamma_a - c_aM_a\frac{\partial \underline{V}_a}{\partial S_a}\end{aligned} \tag{5b}$$

$$\underline{V}_aM_a = -(c_a\underline{V}_a - \sigma_a)\,\delta_pN_p\frac{\partial P_p}{\partial c_a} \tag{5c}$$

$$(c_b\underline{V}_b - \sigma_b)\,N_s\delta_s\frac{\partial P_s}{\partial S_s} = \gamma_s \tag{5d}$$

$$\begin{aligned}(c_b\underline{\underline{V}}_b - \sigma_b)N_s\delta_s \frac{\partial P_s}{\partial \underline{\underline{V}}_b}\frac{\partial \underline{\underline{V}}_b}{\partial S_b} \\ - (c_b\underline{\underline{V}}_b - \sigma_b)\,(N_sP_s + \bar{z}_s)G(V_b)^{n_b}\ln G(V_b)\frac{\partial n_b}{\partial S_b} \\ + c_bM_b\frac{\partial \underline{\underline{V}}_b}{\partial S_b} = \gamma_b\end{aligned} \tag{5e}$$

$$\underline{\underline{V}}_bM_b = -\,(c_b\underline{\underline{V}}_b - \sigma_b)\,\delta_sN_s\frac{\partial P_s}{\partial c_b}. \tag{5f}$$

The left sides of the above equations are the marginal returns with respect to a decision variable, whereas the right sides are marginal costs. Equations (5a) and (5d) equate the marginal returns from search for buy and sell orders to the marginal costs of the respective searches. Equations (5b) and (5e) equate the marginal returns from search for more favorable bids and asks to the marginal costs associated with these search activities. The marginal return from search for ask quotes, represented on the left side of (5b), includes a revenue increase associated with an increase in market share P_p as a result of a lower expected minimum ask price and a revenue increase associated with an increase in the probability of a match. The second term is positive because $\ln(1 - F(V_a))$ is negative. Similarly, the marginal return from search for bid quotes in (5e) includes these two revenue components plus a revenue increase that results from a higher expected transaction price $\underline{\underline{V}}_b$. The marginal cost includes the cost of time and a sacrifice of commission revenue resulting from a lower transaction price in the case of purchase, according to Proposition 1. Brokers are willing to search for the best prices for their customers because the probability of attracting customers depends on their ability to execute orders at the most favorable prices. By (A7), $\partial P_p/\partial \underline{V}_a < 0$, and $\partial P_s/\partial \underline{\underline{V}}_b > 0$. Equations (5c) and (5f) equate the marginal revenue of increasing the commission rate to the marginal cost of the corresponding loss of purchase and sell volume (or market share).

Equations (5a), (5b), (5d), and (5e) can be solved for the optimal levels of different search activities, while (5c) and (5f) can be used to determine the broker's optimal commission rate. The level of search for customers and commission rates will then determine the expected market share for the broker, as measured by P_p and P_s.

The broker's search activities consist of advertising, contacting clients, and acquiring and disseminating market information. According to (A6), the return from a broker's search effort increases at a decreasing rate:

$$\frac{\partial P_p}{\partial S_p} > 0, \qquad \frac{\partial P_s}{\partial S_s} > 0, \qquad \frac{\partial^2 P_p}{\partial S_p^2} < 0, \qquad \frac{\partial^2 P_s}{\partial S_s^2} > 0.$$

However, the productivity of search may also depend on the broker's reputation, experience, and office size, as well as on competition from other brokers. Given a search production function, each broker will ultimately arrive at an optimal resource allocation.

III. BROKERAGE MARKET EQUILIBRIUM

In this section, we examine equilibrium conditions by deriving the demand and supply functions for brokerage services. Let there be N brokers. The total number of purchase orders matched is

$$M_{\mathrm{A}} = \sum_{i=1}^{N} M_{\mathrm{a}i} = \sum_{i=1}^{N} \delta_{\mathrm{p}i}^{*} (N_{\mathrm{p}} P_{\mathrm{p}i}^{*} + \bar{z}_{\mathrm{p}i}). \tag{6}$$

Similarly, the total number of sell orders matched is

$$M_{\mathrm{B}} = \sum_{i=1}^{N} M_{\mathrm{b}i} = \sum_{i=1}^{N} \delta_{\mathrm{s}i}^{*} (N_{\mathrm{s}} P_{\mathrm{s}i}^{*} + \bar{z}_{\mathrm{s}i}). \tag{7}$$

The subscript i is now added to show the unique value for broker i, and the asterisk is used to indicate the optimal levels of variables. The variables M_{A} and M_{B} represent the supply of brokerage services to buyers and sellers, respectively. Both M_{A} and M_{B} are functions of commission rates.

The demand for brokerage services equals the number of purchase and sell orders, N_{p} and N_{s}, arriving in a given time period, multiplied by the expected market shares held by brokers. Denote the expected market shares of brokers as ρ_{p} and ρ_{s} for purchase and sell orders. Then $\rho_{\mathrm{p}} = \sum_i P_{\mathrm{p}i}^{*}$ and $\rho_{\mathrm{s}} = \sum_i P_{\mathrm{s}i}^{*}$. The values of ρ_{p} and ρ_{s} depend on the commission rates, c_{a} and c_{b}. The demand for brokerage services by buyers and sellers can be expressed as $\rho_{\mathrm{p}} N_{\mathrm{p}}$ and $\rho_{\mathrm{s}} N_{\mathrm{s}}$, respectively. The demand function thus measures the expected number of orders to be consummated.

The Walrasian market clearing conditions are

$$M_{\mathrm{A}} = \rho_{\mathrm{p}} N_{\mathrm{p}} \tag{8}$$

$$M_{\mathrm{B}} = \rho_{\mathrm{s}} N_{\mathrm{s}}. \tag{9}$$

The broker's commission and the expected number of orders matched in equilibrium are determined by (8) and (9). The following proposition summarizes the brokerage market equilibrium.

PROPOSITION 2. *Both the demand for and supply of brokerage services are downward-sloping functions of the endogenously determined equilibrium commission rates.*

Proof. To show that both the demand and the supply functions are downward sloping, we differentiate these functions with respect to the commission rate to obtain

$$\frac{\partial M_{\mathrm{A}}}{\partial c_{\mathrm{a}}} = N_{\mathrm{p}} \sum_{i}^{N} \delta_{\mathrm{p}i}^{*} \frac{\partial P_{\mathrm{p}i}^{*}}{\partial c_{\mathrm{a}}} < 0 \tag{10a}$$

$$\frac{\partial \rho_{\mathrm{p}} N_{\mathrm{p}}}{\partial c_{\mathrm{a}}} = N_{\mathrm{p}} \frac{\partial \rho_{\mathrm{p}}}{\partial c_{\mathrm{a}}} = N_{\mathrm{p}} \sum_{i}^{N} \frac{\partial P_{\mathrm{p}i}^{*}}{\partial c_{\mathrm{a}}} < 0 \tag{10b}$$

$$\frac{\partial M_{\mathrm{B}}}{\partial c_{\mathrm{b}}} = N_{\mathrm{s}} \sum_{i}^{N} \delta_{\mathrm{s}i}^{*} \frac{\partial P_{\mathrm{s}i}^{*}}{\partial c_{\mathrm{b}}} < 0 \tag{11a}$$

$$\frac{\partial \rho_{\mathrm{s}} N_{\mathrm{s}}}{\partial c_{\mathrm{b}}} = N_{\mathrm{s}} \frac{\partial \rho_{\mathrm{s}}}{\partial c_{\mathrm{b}}} = N_{\mathrm{s}} \sum_{i}^{N} \frac{\partial P_{\mathrm{s}i}^{*}}{\partial c_{\mathrm{b}}} < 0. \tag{11b}$$

The equilibrium commission rate is determined where supply equals demand. ■

One striking feature of this proposition is that the brokerage service supply function is decreasing in the commission rate. This relationship holds because brokers will lose their clients if they increase their commission rates. Thus, an increase in the commission rate reduces the number of orders matched and, hence, the supply of brokerage services to buyers and sellers. Also, the supply of and demand for brokerage services are interdependent, in that a change in any of the parameters in the model affects both demand and supply simultaneously.

Figure 1 portrays the brokerage market equilibrium conditions for purchase and sell orders. The equilibrium commission rates for buy and sell

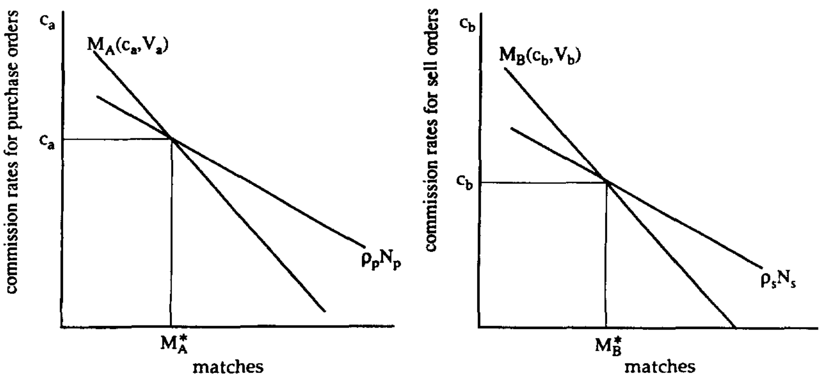

FIG. 1. The brokerage market equilibrium conditions for purchase and sell orders.

orders are not necessarily equal in the short run. Each equilibrium commission rate is determined by separate brokerage demand and supply equilibrium conditions for buy and sell orders. For example, the probability of matching buy orders may be higher than the probability of matching sell orders. This difference could result from the existence of more buyers who believe that the current ask prices are low relative to their purchase reservation prices than there are sellers who believe that the current bids are high relative to their sell reservation prices. In this situation, the equilibrium commission rate for buy orders will be lower than that for sell orders. Likewise, the equilibrium number of purchase orders matched by brokers (M_A) may not equal the equilibrium number of sell orders matched by brokers (M_B). As shown in Fig. 1, this phenomenon can occur because it is sometimes more difficult for brokers to fill sell orders than buy orders (or vice versa) due to different brokerage demand and supply conditions for these two types of orders. The fact that M_A is not equal to M_B does not imply that total quantity of securities supplied is not equal to total quantity demanded or that the securities market is out of equilibrium, because some investors may trade directly with dealers.

Equal commission rates for executing purchase and sell orders occur when $\rho_p N_p = \rho_s N_s$ and $M_A = M_B$. The former condition holds if order flows are balanced ($N_p = N_s$) and the broker's shares of purchase and sell orders are equal ($\rho_p = \rho_s$). This condition may hold in a long-run equilibrium in which purchase and sell order flows are equal and brokers establish stable customer relationships such that customers tend to consolidate their purchase and sell orders with the same broker. If the dealers' inventory level remains constant (equal to their desired inventory level) in the long run, total matches for purchase orders M_A must equal total matches for sell orders M_B. Of course, dealers may need to adjust their bids and asks to balance order flows and to reach their desired inventory levels. Therefore, the equality of commission rates for buying and selling securities would be consistent with a long-run equilibrium.

The equilibrium conditions displayed in Fig. 1 are contingent on a given distribution of bid and ask prices, which, in turn, depends on dealers' portfolio positions. The interactions between brokerage and dealer service markets are explored next.

IV. INTERACTIONS BETWEEN BROKERAGE AND DEALER SERVICE MARKETS

In this section we consider the interaction between brokerage and dealer service markets. For derivations of the demand and supply of dealer services, see Copeland (1976), Epps (1976), Stoll (1978), Cohen *et al.* (1979, 1981), Amihud and Mendelson (1980), and Ho and Stoll (1980, 1981, 1983).

The M_A and M_B derived earlier constitute part of the market demand for, and supply of, securities. The remaining portion of the market consists of investors' direct transactions with dealers. Denote this remaining portion of the securities market demand and supply M_A^d and M_B^d. Then $Q_A = M_A + M_A^d$ and $Q_B = M_B + M_B^d$ are the total demand and supply functions for securities.

Dealers cover the cost of immediacy by purchasing the securities at a price lower than the true market price (bid) and selling at a price higher than the true market price (ask). The true price is defined as the unobservable market equilibrium price in the absence of transaction costs (see Demsetz, 1968). Assume that the dealer's spread is established initially at $\bar{V}_a^* - \bar{V}_b^*$, as shown in Fig. 2. The equilibrium ask price is given by the intersection of dealers' supply curve D_A and the market demand for securities Q_A. Similarly, the equilibrium bid price is determined by the intersection of dealers' demand curve D_B and the market supply of securities Q_B.

In Fig. 2, $\bar{V}_a$ and $\bar{V}_b$ (labeled on the vertical axis) are the average ask and bid prices, which can be best described as the prices quoted by a representative dealer.[5] If, for some reasons, the efficiency of matching the orders by brokers increases such that Q_A and Q_B increase for a given search effort, then the bid–ask spread will be widened.[6] As shown in Fig. 2, the spread increses to $\bar{V}_a' - \bar{V}_b'$ as Q_A and Q_B shift to the right.[7]

The demand and supply functions for dealer service in Fig. 2 are consistent with Stoll (1978) and Ho and Stoll (1983). The bid–ask spread increases as the trading volume for a security increases because dealers incur higher holding costs and are forced to maintain portfolios that are likely to deviate farther from their original desired positions.

The bid and ask price distributions established in the dealers' market also have an impact on the equilibrium conditions of the brokerage market. A reduction in the ask price by some dealers will raise the value of $F(V_a)$, which in turn raises the market supply of broker services. On the other hand, a reduction in the ask price also encourages buyers to enter the market. Similarly, an increase in the bid price by some dealers will increase both the market supply of and demand for broker service to

[5] Note that $\bar{V}_a$ and $\bar{V}_b$ are different from $\underline{V}_a$ and $\underline{\underline{V}}_b$. The latter are expected minimum ask and maximum bid prices resulting from brokers' search activities.

[6] The efficiency of matching orders increases as the number of directly competing market makers increases, because the number of ask (bid) quotes n_a (n_b) may rise as the number of competing market makers increases. Garbade and Silber (1976, p. 725) have indicated that dealers' price quotations may differ for reasons such as changes in market conditions, differences in transaction cost functions (related to volume) and inventory policies, supply and demand fluctuations, heterogeneous expectations, and the obsolescence of information.

[7] There is no need to force M_A to be equal to M_B in this particular case. However, if dealers wish to maintain constant inventory, then their demand and supply will need to adjust.

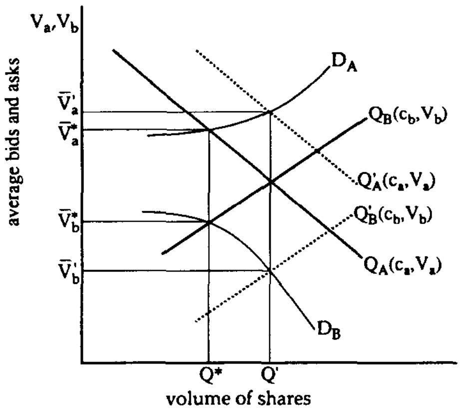

FIG. 2. The effects of broker matches on the bid–ask spread.

sellers. More specifically,

$$\frac{\partial M_A}{\partial \overline{V}_a} = \sum_{i=1}^{N} (N_p P^*_{pi} + \bar{z}_{pi}) \frac{\partial \delta^*_{pi}}{\partial \overline{V}_a} + \sum_{i=1}^{N} \delta^*_{pi} P^*_{pi} \frac{\partial N_p}{\partial \overline{V}_a} < 0 \tag{12a}$$

$$\frac{\partial \rho_p N_p}{\partial \overline{V}_a} = \rho_p \frac{\partial N_p}{\partial \overline{V}_a} < 0 \tag{12b}$$

$$\frac{\partial M_B}{\partial \overline{V}_B} = \sum_{i=1}^{N} (N_s P^*_{si} + \bar{z}_{si}) \frac{\partial \delta^*_{si}}{\partial \overline{V}_b} + \sum_{i=1}^{N} \delta^*_{si} P^*_{si} \frac{\partial N_s}{\partial \overline{V}_b} > 0 \tag{13a}$$

$$\frac{\partial \rho_s N_s}{\partial \overline{V}_b} = \rho_s \frac{\partial N_s}{\partial \overline{V}_b} > 0. \tag{13b}$$

The effect of dealers' prices on the supply of broker services is larger than that on the demand for broker services for most reasonable exogenous parameter values in the model.[8] Figure 3 displays the effect of average bid

[8] The effect on the supply is very likely to be larger than that on the demand, if market orders are the major type of orders received by the broker, or if the number of bid and ask quotes is large. Under either of these two conditions, δ^* will be close to one and the second term on the right side of (12a) or (13a) will be close to (12b) or (13b). Even if the above conditions do not hold, it will still be very likely that the effect on the supply outweighs that on the demand. To see this, assume that δ^*_{pi} is the same for each broker and $\bar{z}_{pi} = 0$. Then rewrite (12a) as

$$\frac{\partial M_A}{\partial \overline{V}_a} = \rho_p N_p \frac{\partial \delta^*_{pi}}{\partial \overline{V}_a} + \rho_p \delta^*_{pi} \frac{\partial N_p}{\partial \overline{V}_a}.$$

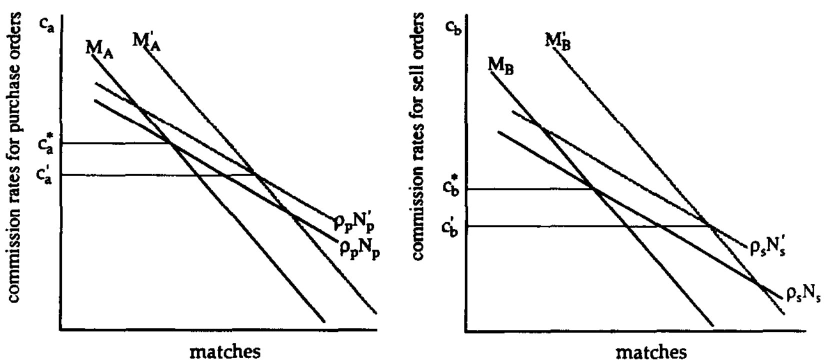

FIG. 3. The effects of bid–ask spreads on broker matches.

and ask prices on the supply of brokerage service as $\overline{V}_b$ increases. Thus, a reduction in the spread in the dealer's market may diminish the broker's commission rate.

The preceding analysis provides a basis for deriving a general equilibrium in brokerage and dealer service markets. Rewrite the brokerage market equilibrium is (8) and (9) as

$$M_A(c_a;\ \overline{V}_a) = \rho_p(c_a)N_p(\overline{V}_a) \tag{8'}$$

$$M_B(c_b;\ \overline{V}_b) = \rho_s(c_b)N_s(\overline{V}_b), \tag{9'}$$

where the arguments in the parentheses are the variables that affect the brokerage demand and supply. Following Stoll (1978) and Ho and Stoll (1983), we specify the bid–ask spread function for a security as

$$s \equiv \overline{V}_a - \overline{V}_b = h(\sigma^2, \lambda, Q(c)), \tag{14}$$

where σ^2 is the variance of security returns, λ relates to a dealer's risk aversion (see (A5)), and Q is transaction volume of the security.[9] The bid–ask spread is positively related to each of these three factors. It is an increasing function of transaction volume, Q, because a dealer incurs larger holding costs by taking a greater position in a security. Furthermore, the adverse information cost increases with Q. Therefore, the higher Q, the larger the bid–ask spread must be to cover the cost of adverse information. In the Stoll and Ho (1983) and Stoll (1978) models,

Then $|\partial M_A/\partial \overline{V}_a| > |\partial \rho_p N_p/\partial \overline{V}_a|$ if $N_p(\partial \delta^*_{pi}/\partial \overline{V}_a) < (1 - \delta^*_{pi})(\partial N_p/\partial \overline{V}_a)$. Since N_p is a large number, this condition is likely to hold.

[9] In our one-period model, we ignore the difference between transaction size and volume.

the bid–ask spread is related to transaction volume under the condition that $Q = Q_A = Q_B$. We adopt this condition in the following analysis.

Differentiating (8′), (9′), and (14) and rearranging terms gives

$$dc_a = \left[\left(-\frac{\partial M_A}{\partial \overline{V}_a} + \rho_p \frac{\partial N_p}{\partial \overline{V}_a}\right) \Big/ \left(\frac{\partial M_A}{\partial c_a} - N_p \rho_p'\right)\right] d\overline{V}_a \tag{15}$$

$$dc_b = \left[\left(-\frac{\partial M_B}{\partial \overline{V}_b} + \rho_s \frac{\partial N_s}{\partial \overline{V}_b}\right) \Big/ \left(\frac{\partial M_B}{\partial c_b} - N_s \rho_s'\right)\right] d\overline{V}_b \tag{16}$$

$$ds = \frac{\partial h}{\partial Q} \frac{\partial Q}{\partial c}\, dc. \tag{17}$$

The term in brackets in (15) is usually positive since $-\partial M_A/\partial \overline{V}_a + \rho_p(\partial N_p/\partial \overline{V}_a) > 0$ as indicated earlier and $\partial M_A/\partial c_a - N_p \rho_p' > 0$ from (10a) and (10b). In contrast, the term in brackets in (16) is usually negative since $-\partial M_B/\partial \overline{V}_b + \rho_s \partial N_s/\partial \overline{V}_b < 0$ from (13a) and (13b) and $\partial M_B/\partial c_b - N_s \rho_s' > 0$ from (11a) and (11b). The relation in (17) is derived under the condition that σ^2 and λ are constant. This constraint allows us to focus exclusively on the effect of trading volume. The change in the average bid–ask spread is related to the change in trading volume, which, in turn, is affected by the commission rate. Since the bid–ask spread represents a round-trip transaction cost, the commission rate used in (17) is also defined in terms of the round-trip commission, where $c = c_a + c_b$.

The following proposition summarizes the main results for the integrated broker and dealer equilibrim.

PROPOSITION 3. *In a general equilibrium in which both the broker and dealer markets are simultaneously in equilibrium, these two markets are interdependent. The partial equilibrium commission rate determined in the brokerage market is positively related to the dealer's bid–ask spread for most reasonable parameter values. The partial equilibrium bid–ask spread determined in the dealer market is negatively related to the broker's commission rate.*

The proof of Proposition 3 is given in the Appendix.

To illustrate the interdependence of broker and dealer markets in a more intuitive way, we present a graphical analysis. First, a change in brokerage market conditions affects the dealer market equilibrium. For example, if the equilibrium c_a drops due to an increase in the efficiency of matching purchase orders, M_A (and Q_A) in Fig. 2 shifts to the right and generates a higher equilibrium $\overline{V}_a$. This result implies a higher value of s. Similarly, if c_b drops due to an increase in the efficiency of matching sell orders, M_B (and Q_B) shifts to the right and produces a lower equilibrium $\overline{V}_b$. Again, the result is that s increases. Therefore, the decrease of either

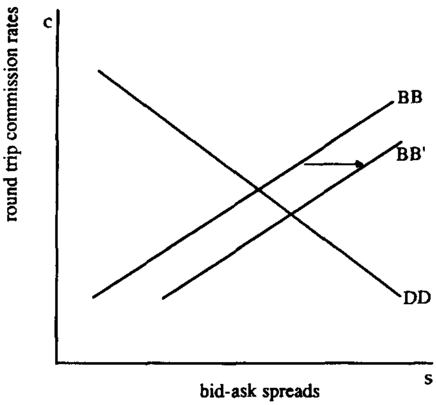

FIG. 4. The equilibrium of brokerage and dealer markets.

c_a or c_b will increase the equilibrium value of s in the dealer service market. On the other hand, a change in dealer market conditions affects the brokerage market equilibrium, as in Fig. 3. For instance, a decrease in $\overline{V}_a$ (a decrease in s) boosts the number of purchase order matches (M_A) and lowers the equilibrium commission rate c_a. Likewise, an increase in $\overline{V}_b$ (a decrease in s) raises the number of sell order matches (M_B) and lowers the commission rate c_b. Thus, c is positively related to dealers' spread s. The lower s, the lower the equilibrium value of c in the brokerage market.

Figure 4 shows the general equilibrium relation between c and s for the broker and dealer service markets. The *BB* and *DD* curves represent the partial equilibrium paths for brokerage and dealer service markets, respectively. At the intersection of these two curves, both markets reach an equilibrium simultaneously.

The values of ρ represent the proportion of total transactions facilitated through broker offices. Recall that $\rho_p = \Sigma P^*_{pi}$ and $\rho_s = \Sigma P^*_{si}$, which measure brokers' shares of purchase and sell orders, respectively. The values of ρ depend on the commission rate. The higher the equilibrium commission rate, the lower the market share held by brokers. Therefore, any factor that affects the equilibrium commission rate will also affect the share of brokered transactions. For example, if the efficiency of matching purchase orders increases, then M_A in Fig. 3 will shift to the right, resulting in a lower commission rate and a higher share of brokered transactions. The increase in match efficiency will then shift the *BB* curve in Fig. 4 to the right. The final equilibrium commission rate will be determined at the intersection of the *BB'* and *DD* curves in Fig. 4. The equilibrium commission rate thus drops and the market share of brokered transactions (ρ_p) increases as the match efficiency increases.

V. EVALUATION OF SOME BROKERAGE SERVICES

In this section, we apply our analysis to two important brokerage services: in-house execution and interdealer brokerage.

1. *In-House Execution Services*

The abolition of NYSE Rule 390 in 1980 allowed brokerage firms to trade stock issues off-board. The purpose of relaxing off-board trading restrictions was to increase competition in security markets. We show that relaxation of off-board trading restrictions could affect brokers' transaction costs and profits.

With in-house execution, brokers need not take their orders to a specialist or a dealer and therefore need not share their commissions with these outside parties. By retaining their orders and executing in house, brokers also reduce handling costs. Thus, in-house execution should be more profitable for brokers.

To analyze in-house execution, we first consider an extreme case in which brokers do not take their own orders to the exchange and where brokers keep their own limit order books. The total match function for broker i becomes

$$M'_{ai} = (N_p P_p + \bar{z}_p)\,[1 - (1 - F(V_a))]^{n'_{ai}}, \tag{18}$$

where n'_{ai} is the number of sell offers received by a broker. In a strictly in-house execution, a broker receives the entire round-trip commission. The expected profit generated by the brokerage service now becomes

$$\pi'_j = (R - \sigma')M_{ai} - \sum_k \gamma_k Sk, \tag{19}$$

where R is the round-trip commission revenue and σ' is the round-trip handling cost. Also, $R = c'\underline{V}$, where $\underline{V}$ is the transaction price. The round-trip commission rate $c' > c_a + c_b$, since the broker does not give up a portion of his/her commission by trading with other dealers or exchange members. Although the commission rates are higher and handling costs are lower, the net effect on the broker's profit in (19) is ambiguous because the number of orders matched M_{ai} will be lower than the results in (2) and (3). Also, the broker will not obtain the most favorable price for buyers and sellers because the execution price is based on the orders received by the broker, which constitute only a subset of the orders flowing into the market. For a large brokerage firm, the effect on M_{ai} and $\underline{V}$ will be small, and the net gain generated by in-house execution is likely to be positive.

However, the case of strictly in-house execution is rare. It is more likely that a broker will try to match as many in-house orders as possible and bring the remaining unmatched orders to the market. In such a case, the match function for a purchase order is

$$M_{ai} = (N_p P_p + \bar{z}_p)\{[1 - (1 - F(V_a))^{n'_{ai}}] + [1 - F(V_a)]^{n'_{ai}} - [1 - F(V_a)]^{n_{ai}}\}, \tag{20}$$

where the first term in the brace represents the probability of in-house execution and the second term represents the probability of outside execution. Note that $n_{ai} > n'_{ai}$, since n_{ai} is the number of available ask quotes in the entire market while n'_{ai} is the number of ask quotes available in the broker's firm.

Similarly, the match function for a sell order will be

$$M_{bi} = (N_s P_s + \bar{z}_s)\{[1 - G(V_b)^{n'_{bi}}] + [G(V_b)^{n'_{bi}} - G(V_b)^{n_{bi}}]\}, \tag{21}$$

where the first term in the brace is the probability of in-house execution and the second term in the brace is the probability of outside execution, and $n_{bi} > n'_{bi}$ for a reason similar to that presented in the previous paragraph.

Although (20) and (21) are identical to (2) and (3), respectively, the broker's profit is not the same as that in (5). Rather, the broker's expected profit now becomes

$$\begin{aligned}\Pi_j^* = {} & R_{1a}(N_p P_p + \bar{z}_p)[1 - (1 - F(V_a))^{n'_{ai}}] \\ & + R_{2a}(N_p P_p + \bar{z}_p)[(1 - F(V_a))^{n'_{ai}} - (1 - F(V_a)^{n_{ai}}] \\ & + R_{1b}(N_s P_s + \bar{z}_s)[1 - G(V_b)^{n'_{bi}}] + R_{2b}(N_s P_s + \bar{z}_s)[G(V_b)^{n'_{bi}} \\ & - G(V_b)^{n_{bi}}] - \sum_k \gamma_k s_k,\end{aligned} \tag{22}$$

where

$$R_{1a} = c'_a \underline{V}_a - \sigma'_a, \qquad R_{2a} = c_a \underline{V}_a - \sigma_a$$

$$R_{1b} = c'_b \underline{\underline{V}}_b - \sigma'_b, \qquad R_{2b} = c_b \underline{\underline{V}}_b - \sigma_b$$

$$c'_a > c_a, \qquad \sigma'_a < \sigma_a, \qquad c'_b > c_b, \qquad \sigma'_b < \sigma_b.$$

Comparing (22) with (5), we see that the expected profit of the broker increases with in-house execution, because $R_{1a} > R_{2a}$ and $R_{1b} > R_{2b}$. In (22), the values of $\underline{V}_a$ and $\underline{\underline{V}}_b$ should be equal to the values of $\underline{V}_a$ and $\underline{\underline{V}}_b$ in (5), because the broker has access to all available market quotes. Furthermore, the broker receives higher buying and selling commissions (c'_a, c'_b) and faces lower handling costs (σ'_a, σ'_b) when the order is executed in house.

Although brokers' expected profits will increase in the short run, the long-run impact of in-house execution is ambiguous. There are at least two potential concerns. First, the revenue received by the exchanges will decline as a result of in-house execution. This reduction in revenue will affect the quality of the ancillary services, which are mostly public-goods-type services, provided by security exchanges. Important services offered by exchanges include supplementary stabilization, surveillance, quality certification of brokerage firms, and the information for price discovery (Cohen *et al.*, 1982). The reduction in these public-goods-type services could affect the order flow and the effective "thickness" and stability of the market. If these problems occur, then the orders matched by brokers as well as total trading volume will decline, an outcome which, according to our analysis, will result in higher commission costs. Second, more profitable in-house execution may encourage brokers to increase their search activities and competition. Higher search efforts by brokers may, to a certain extent, represent competition among brokers for a given number of potential orders. The increase in brokers' search efforts may not result in higher transaction volume, so that the effect may be simply to increase the cost of brokerage services. Furthermore, a broker's own execution is usually not as efficient as the centralized market execution. For example, it generally takes a longer time for a broker with a smaller order flow to match an order in house. Also, higher competition among brokers reduces the economies of scale in collecting information. Thus, in-house execution tends to increase brokers' transaction costs.

2. *Interdealer Brokerage*

Dispersion of price quotations in a dealer market often leads to a high cost of searching for favorable quotations. This result holds particularly with respect to government bond markets, where large flows of public orders take place periodically. The fluctuations of dealer inventories due to uneven order flows are primarily responsible for the dispersion of quotations. Interdealer trading facilitates expeditious inventory adjustment and reduces the dispersion of dealers' quotations. However, search for the most favorable bids and asks can be time consuming. The relatively inefficient search by dealers has stimulated the development of interdealer brokerage.

An interdealer broker provides several specialized services (Maru and Takahashi, 1985). First, the interdealer broker provides the most favorable bid and ask prices to dealers after acquiring interdealer quotes. This search service allows dealers to quickly adjust their inventory positions to desired levels and to improve their own quotations. Second, the interdealer broker searches and negotiates with a compatible trading partner for a dealer. Third, the interdealer broker conceals information about the transaction. This concealment is particularly desirable in the block trading of equities from the traders' viewpoint.

Interdealer brokerage reduces the cost of transacting. An interdealer broker provides pooling services for dealers and also increases price information for public investors. In doing so, the broker improves the efficiency of transactions and reduces liquidity costs by exploiting economies of scale in assembling information.[10]

The interdealer broker provides a centralized matching service. The broker collects all available quotes and communicates the best bids and asks to dealers. Instead of obtaining information through costly bilateral communication, dealers now can instantly acquire the most favorable quotes through a broker. This centralization results in search cost savings in the dealer market. Before the development of interdealer brokerage, the match function characterizing the buyer's search could be expressed as

$$\delta = \begin{cases} 1 - (1 - F(V))^n & \text{if } V \geq V_a^{\min} \\ 0 & \text{if } V < V_a^{\min}, \end{cases} \tag{23}$$

where n is the number of quotes searched by a buyer, $V_a^{\min}$ is the minimum of ask quotes, and V is the reservation price. The number of quotes searched by a buying dealer depends on the cost of additional search and on the order size.[11] The cost of searching for favorable quotes is higher when the inventory positions of individual dealers change rapidly due to a heavy flow of public orders and when the bilateral communication between dealers is slow.[12] The lower n, the lower the probability of a match.

In contrast, in the case of interdealer brokerage, the distribution of ask prices degenerates to one point, the most favorable ask $V_a^{\min}$. If the most favorable ask is lower than the buyer's reservation price, then the match

[10] The interdealer broker can economize on the costs of gathering information on the quotes of dealers for a particular security and can provide the best bid and ask prices to each dealer at lower costs.

[11] We can obtain the optimal n by setting the absolute value of (A.3) in the Appendix equal to the cost of additional search. It can be shown that a higher cost of additional search leads to a lower level of the buyer's search since the marginal gain of search declines with n. Note that the marginal gain of search in (A.3) is expressed in terms of one share. The total additional gain per order is equal to the units of security in an order times the absolute value of (A.3). Thus the larger the order size, the greater the marginal gain of search and the level of the buyer's search.

[12] To see how large n can be, assume that a dealer's inventory position changes every two minutes. If there are 40 dealers in the bond market and it takes 15 seconds to contact a competing dealer, then the buying delaer can contact only 8 of the 40 dealers in two minutes in order to restore the desired inventory level.

is completed. Otherwise, the buyer must wait. More specifically,

$$\delta = \begin{cases} 1 & \text{if } V \geq V_a^{\min} \\ 0 & \text{if } V < V_a^{\min}. \end{cases} \tag{24}$$

Thus, interdealer brokerage increases the probability of match, as shown by a comparison of (24) with (23). This gain is larger when n is smaller (as $n \to \infty$, (23) $\to$ (24)). This result holds particularly when the search cost is high. According to the analysis in the preceding section, the increase in match probability will increase the trading volume and reduce the commission fee.

VI. THE DETERMINANTS OF BROKERAGE COMMISSIONS

The search model allows us to identify a number of determinants of the brokerage commission rate. These include the order cost, the number of competing market makers, the order size, and the risk and unit price of the security. Some of these determinants have been recognized in previous empirical studies. For instance, order size (Blum and Lewellen, 1983), stock price (West and Tinic, 1971; Ofer and Melnick, 1978), and the number of dealers making markets (West and Tinic, 1971) are found to be significantly related to the commission rate. However, no theoretical model has been offered to explain the directional effects of these variables. In the following analysis, we apply our model to discover the relationship between the brokerage commission rate and its determinants.

1. *Order Cost*

The order cost affects the marginal return of brokers' order-matching activity. A decrease in the order cost per share encourages brokers to search because the net marginal return is increased. Thus, a decrease in the order cost increases the number of matches for both sell and purchase orders. As the number of orders matched increases, the supply of brokerage service increases and the equilibrium commission rate drops.[13]

2. *The Number of Competing Market Makers*

A rise in the number of competing market makers usually increases the available quotes of bids and asks and, therefore, increases the gain from search and the probability of match. However, the magnitude of this effect depends on the number of pre-existing quotes. For instance, if the number of competing quotes increases such that n_a increases from n_1 to

[13] The increase in matches is largely due to an increase in the probability of a match and to a more intensive search by brokers. As indicated earlier, the effects on P_p and P_s are likely to be negligible for the brokerage industry as a whole.

n_2, the increase in the total number of purchase order matches for a broker is $\Delta M_a = (P_p N_p + \bar{z}_p)[(1 - F(V_a))^{n_1} - (1 - F(V_a))^{n_2}]$. The magnitude inside the brackets depends on n_1 and n_2. If n_1 is already very large, then the increase in the number of competing quotes has only a negligible effect on the number of matches. On the other hand, if n_1 is small, then ΔM_a will be relatively large. Thus, an increase in the number of dealers (e.g., due to a market integration) increases the number of order matches and decreases the equilibrium commission rate. These effects will be larger for securities with fewer dealers.

3. *Order Size*

In our search model, the size of an order is restricted to one share for simplicity. We can relax this assumption to allow order size to vary across customers. Since the per-transaction marginal return to a broker is positively related to order size, an increase in order size increases the broker's optimal search effort and the supply of matches and reduces the equilibrium commission rate.

4. *Security Risk*

The bid–ask spread is positively related to security risk (Ho and Stoll, 1983). The potential capital loss from holding a risky security causes dealers to raise ask prices and lower bid prices in order to compensate for risk bearing. According to the preceding analysis, a larger bid–ask spread reduces the probability of match for both purchase and sell orders and therefore increases the equilibrium commission rate. Thus, the commission rate will be positively related to the risk of a security.

5. *Unit Price of Securities*

For any given order size, the higher the price of a security, the greater the marginal return per transaction. As is true of the effect of order size, a higher security price increases broker search and the probability of match and reduces the equilibrium commission rate.

In summary, variations of commission rates for securities can be explained by transaction and asset characteristics. In general, the commission rate will be an increasing function of the order cost and the risk of the security and a decreasing function of the number of competing market makers, the unit price of the security, and the order size.

VII. CONCLUSIONS

In this paper, a search model is developed to describe the behavior of securities brokers in a world with random prices. The model is used to analyze a variety of issues related to broker services. The analysis concludes that the equilibrium commission rate is affected by dealers' positions and by other factors such as the order cost, the number of competing market makers, the search efficiency, the order size, and the risk and unit price of the security. The paper also shows that equilibrium in the dealer market is affected by conditions in the brokerage market and vice versa.

Several brokerage market issues are inadequately addressed in this paper. First, the search functions and the frequency distributions of bids and asks are not specified. Second, the effect of information flow on broker search is neglected. Intuitively, the pattern of information release affects the waiting time. This remains to be explored.

This paper nevertheless provides numerous implications concerning the commission rate and brokerage market structure. The analysis predicts that the commission rate is positively related to order costs and security risk and negatively related to search efficiency, order size, price, and the number of competing market makers for a particular security. Moreover, the commission rate is predicted to be lower when the bid–ask spread of a security is smaller. It is also shown that in-house services increase brokers' profits, whereas interdealer brokerage reduces the dispersion of dealers' price quotations and transaction costs and increases trading volume. Thus, the paper provides a theoretical foundation for determining the commission rate structure and for evaluating different brokerage systems.

APPENDIX

A. *The Effect of Broker's Search Level on $\underline{V}_a$ and $\underline{\underline{V}}_b$*

Following Stigler (1961), the density function of the minimum ask prices is specified as

$$f(V_a) = n_a(1 - F(V_a))^{n_a - 1}F'(V_a). \tag{A.1}$$

The expected minimum ask price is then equal to

$$\underline{V}_a = E(V_a/n_a) = \int V_a n_a(1 - F(V_a))^{n_a - 1}F'(V_a)\, dV_a. \tag{A.2}$$

The expected minimum price can be shown as a decreasing function of n_a; that is,

$$\frac{\partial \underline{V}_a}{\partial n_a} = \int V_a(1 - F(V_a))^{n_a - 1}F'(V_a)(1 + n_a \ln(1 - F(V_a))\, dV_A < 0. \tag{A.3}$$

The proof of (A.3) involves stochastic comparison. Let $F_n = 1 - (1 - F(V_a))^n$ and $F_{n+1} = 1 - (1 - F(V_a))^{n+1}$. Then $F_{n+1} > F_n$ corresponds to $E(V_a/n) > E(V_a/n + 1)$.

The effect of search on the expected minimum price is

$$\frac{\partial \underline{V}_a}{\partial S_a} = \frac{\partial \underline{V}_a}{\partial n_a}\frac{\partial n_a}{\partial S_a} < 0, \qquad \text{if } \frac{\partial n_a}{\partial S_a} > 0. \tag{A.4}$$

Similarly, it can be shown that the expected maximum bid is

$$\underline{\underline{V}}_b = E(V_b/n_b) = \int V_b n_b G(V_b)^{n_b - 1} G'(V_b)\, dV_b. \tag{A.5}$$

Let $G_n = G(V_b)^n$ and $G_{n+1} = G(V_b)^{n+1}$. Then $G_n > G_{n+1}$ implies that $E(V_b/n) < E(V_b/n + 1)$. Or, in continuous terms,

$$\frac{\partial \underline{\underline{V}}_b}{\partial S_b} = \int V_b G(V_b)^{n_b - 1} G'(V_b)(1 + n_b \ln G(V_b))\, dV_b > 0. \tag{A.6}$$

Furthermore, the effect of search on the bid price is

$$\frac{\partial \underline{\underline{V}}_b}{\partial S_b} = \frac{\partial \underline{\underline{V}}_b}{\partial n_b}\frac{\partial n_b}{\partial S_b} > 0, \qquad \text{if } \frac{\partial n_b}{\partial S_b} > 0. \tag{A.7}$$

B. *Proof of Proposition* 3

From (17), it is clear that $ds/dc < 0$ since $\partial h/\partial Q > 0$ and $\partial Q/\partial c < 0$, indicating that the bid–ask spread is negatively related to the commission rate. To show the relation of the broker's commission with the dealer's bid–ask spread, we sum (15) and (16) to give

$$\begin{aligned} dc = {} & \left[\left(-\frac{\partial M_A}{\partial \overline{V}_a} + \rho_p \frac{\partial N_p}{\partial \overline{V}_a}\right)\Big/\left(\frac{\partial M_A}{\partial c_a} - N_p \rho_p'\right)\right] d\overline{V}_a \\ & + \left[\left(-\frac{\partial M_B}{\partial \overline{V}_b} + \rho_s \frac{\partial N_s}{\partial \overline{V}_b}\right)\Big/\left(\frac{\partial M_B}{\partial c_b} - N_s \rho_s'\right)\right] d\overline{V}_b, \end{aligned} \tag{B.1}$$

where $dc = dc_a + dc_b$ is positively related to $d\overline{V}_a$ and negatively related to $d\overline{V}_b$ for most reasonable parameter values in the model. For a given bid price $\overline{V}_b$, an increase in the ask price, $d\overline{V}_a > 0$, causes an increase in the bid–ask spread s and in the round-trip commission rate c. Similarly, for a given ask price $\overline{V}_a$, a decrease in the bid price, $d\overline{V}_b < 0$, causes an increase in the bid–ask spread s and in the round-trip commission rate c. Thus, the round-trip brokerage commission rate is positively related to the bid–ask spread. Finally, the partial equilibrium related in the dealer market, (17), and the partial equilibrium relation in the brokerage market, (B.1), jointly determine the general equilibrium average bid–ask spread and commission rate.

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